Four-Current ~~Tensor~~ representation

OR Electromagnetic four-potential (see Wiki)

and Lagrange.

$$F_{\mu\nu} \equiv \begin{pmatrix} 0 & E_x & E_y & E_z \\ & 0 & B_z & -B_y \\ -* & & 0 & B_x \\ & & & 0 \end{pmatrix}$$

$$F^{\mu\nu}(E, B) = F_{\mu\nu}(-\vec{E}, \vec{B})$$

So $\underbrace{F_{\mu\nu} F^{\mu\nu}} \propto \underline{\vec{B}^2 - \vec{E}^2 = (B-E)(B+E)}$

A lot of interesting properties in this eg

$$x^2 - y^2 = x'^2 - y'^2$$

$$x_1^2 - x_2^2 = x_1'^2 - x_2'^2$$

$$\Leftrightarrow \quad x_i' = U_{ij} x_j$$

$$x_1'^2 - x_2'^2 = U_{1j} U_{1k} x_j x_k - U_{2j} U_{2k} x_j x_k$$

$$= (U_{1j} U_{1k} - U_{2j} U_{2k}) x_j x_k \Rightarrow (U^T \sigma_z U)_{jk} \underline{x_j x_k}$$

$$= x_1^2 - x_2^2 \quad \boxed{\cancel{U_{1j} U_{2k} = \sigma^z_{jk}}} \text{ 자의}$$

$\boxed{U^T \sigma_z U = \sigma_z}$

OR $\boxed{\cancel{U^T U = \sigma_z}}$ } ~~det(U) = 1 (?)~~

$\boxed{U_{ij} \in \text{Real}}$

From field.

Def in Baidu Baike (百度.百科)

$g(\vec{x})$ $\quad T(\vec{x})$ 标量 scalar field

$$\left.\begin{array}{l}\vec{B}(\vec{x})\\ \vec{E}(\vec{x})\\ \vec{g}(\vec{x})\end{array}\right\}$$ 矢量 vector

material interact with field to eliminate the non-local / long-range interaction.

Coulomb Interation
$$\begin{cases} \dfrac{q_1 q_2}{|\vec{r}_1-\vec{r}_2|} \equiv q_2 \phi \\ \phi = \dfrac{q_1}{|\vec{r}_1-\vec{r}_2|} \rightarrow \text{potential} \end{cases}$$

Gravitional Force

$$\left(\frac{GMm}{r^2}\right) = mg$$

$g = \left(\dfrac{GM}{r^2}\right)$ is gravitational field.

No non-local Interaction.

Lagrange for EM:

How to remember it.

$$\left\{\begin{array}{l} \nabla\cdot\vec{E}=\rho \\ \nabla\times\vec{B}=0 \end{array}\right.\Bigg\}\ \text{Gauss law.}$$

$$\left.\begin{array}{l} \nabla\times\vec{E}-\frac{\partial\vec{B}}{\partial t}\equiv 0 \\ \nabla\times\vec{B}+\frac{\partial\vec{E}}{\partial t}\equiv\vec{J} \end{array}\right.$$

Faraday, 变化磁场产生电场.

① $\int\vec{A}\cdot d\vec{l}\equiv\oint\vec{B}\cdot d\vec{s}\equiv\int(\nabla\times\vec{A})\cdot d\vec{s}\equiv\Phi$

② Any vector may be written as

$$\boxed{\vec{V}=\nabla\times\vec{A}+\nabla\phi}$$

so if $\vec{A}=\nabla\phi$

$\Leftrightarrow$ $\int\nabla\phi\cdot d\vec{l}=\int\underline{\nabla\times\nabla\phi}\cdot d\vec{s}\equiv 0$

$\int\vec{B}\cdot d\vec{l}\equiv\int(\nabla\times\vec{A})\cdot d\vec{l}\equiv ?$ Biot-Savart law

$$\vec{B}\equiv\oint\frac{\mu_0 I}{4\pi}\frac{d\vec{l}\times\vec{e}_r}{r^2}\qquad \vec{e}_r=\vec{r}/|\vec{r}|=\vec{r}/r$$

$$\frac{\vec{r}}{r^3}=\left(\nabla\frac{1}{r}\right)\qquad \equiv\oint\frac{\mu_0 I}{4\pi}\frac{d\vec{l}\times\vec{r}}{r^3}$$

$$\boxed{\nabla\cdot\vec{B}\equiv 0}$$

related to linking #

Ampere circuital thm

$$\underline{\nabla \times \vec{B} = \mu_0 \vec{J}}$$

$$\nabla \cdot (\nabla \times \vec{B}) \equiv 0 \Rightarrow \nabla \cdot \vec{J} = 0 \Rightarrow \boxed{\partial \rho / \partial t \equiv 0}$$

两个为0的算式

$$\begin{cases} \nabla \times \nabla \phi \equiv 0 & \text{梯无旋} \\ \nabla \cdot (\nabla \times \vec{B}) \equiv 0 & \text{旋无散} \end{cases}$$

$\underline{\nabla \cdot (\nabla \times \vec{B})}$ <u>is scalar field</u>

$$\nabla \cdot (\nabla \times \vec{B})$$

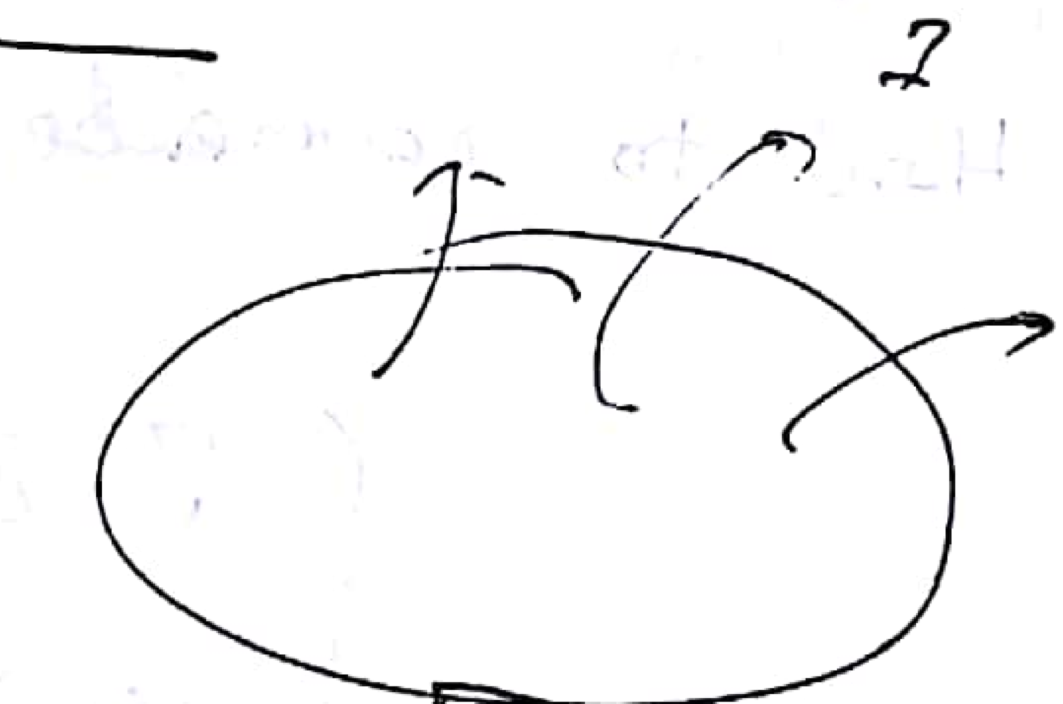
$$\partial_i \underbrace{(\nabla \times \vec{B})}^i$$

$$\partial_i \underbrace{\varepsilon^{ijk} \partial_j B_k}$$

$$= \varepsilon^{ijk} \underline{\partial_i \partial_j} B_k \equiv \boxed{0}$$

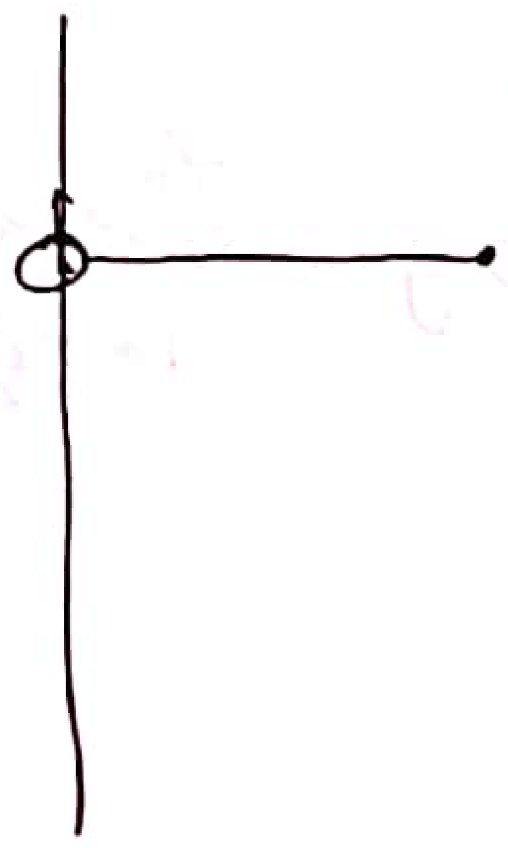
可交换

Ampere circuital law/theorem



$$\boxed{\oint \vec{B}\cdot d\vec{l} = \mu_0 I}$$

$\Rightarrow$ 從 B 時 $\oint (\nabla \times \vec{B})\cdot d\vec{s} = \int \mu_0 \vec{j}\cdot d\vec{s} \Rightarrow \boxed{\nabla \times \vec{B} = \mu_0 \vec{j}}$



$$B \cdot 2\pi r = \mu_0 I \Rightarrow B = \left(\frac{\mu_0 I}{2\pi r}\right).$$

eg $\begin{cases} r = 1\ \text{cm} \\ I = 1\ \text{Å} \end{cases}$ 问 $B = ?$

$\mu_0 = 1.25 \times 10^{-6}\ H/m$

$I = 1\ A$

$$B = \frac{1.25 \times 10^{-6}\ H\cdot A/m}{6 \cdot \times 10^{-2}\ m}$$

$= 10^{-9}\ (H \cdot A/m^2)$

$= 10^{-9}$ Tesla

Field eq & Maxwell eq

1: The concept was first seen in Maxwell eq.

2: E-M enerery

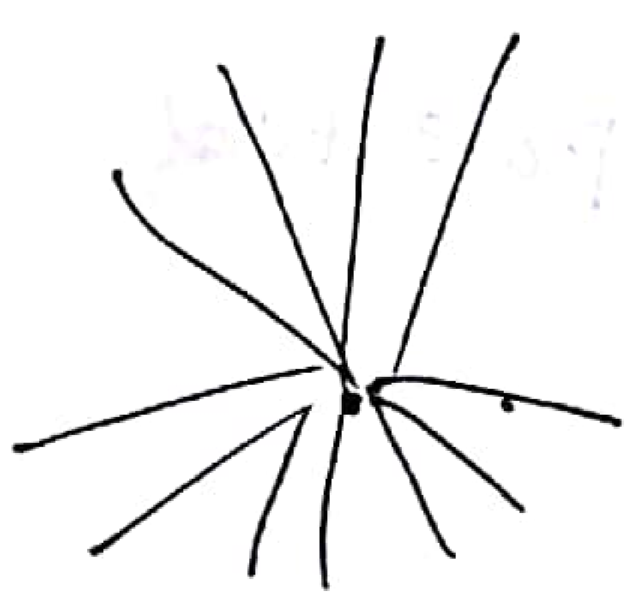
$$E \propto \int (\vec{E}^2 + \vec{B}) \, d\vec{x}$$

So the E-M energy is stored spatially.

3: Estimation of electron radius $E = mc^2$

$$U = \frac{e^2}{4\pi\varepsilon_0 r}$$

$$\vec{E} = -\nabla U = -\left(\frac{e^2 \vec{r}}{4\pi\varepsilon_0 |r|^3}\right).$$



$$\int \varepsilon_0 \vec{E}^2 \, d^3\vec{x}$$

$$= \int_{r_0}^{+\infty} \varepsilon_0 \frac{e^2 \, r^2}{(4\pi\varepsilon_0)^2 \, r^6} \cdot 4\pi r^2 \, dr$$

$$= \frac{e^2}{(4\pi)\varepsilon_0} \left(\int_{r_0}^{+\infty} \frac{1}{r^2} \, dr \right)$$

$$\sim \left(\frac{e^2}{4\pi\varepsilon_0 r_0} \right) = mc^2 = 0.511 \times 10^6 \, eV$$

$$\frac{e^2}{4\pi\varepsilon_0 a_B} = 13.6 \, eV$$

$$\Leftrightarrow \quad \frac{a_B}{r_0} = \frac{0.511 \times 10^6}{13.6} \simeq \frac{5.11 \times 10^4}{1.36} \sim 3 \times 10^4$$

$$\boxed{r_0 \sim 10^{-4} a_B}$$